

Revised Final Report on Reliability  
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15P

## ABSTRACT

In the design phase of a system, how does a design engineer or manager choose between a subsystem with .990 reliability and a more costly subsystem with .995 reliability? When is the increased cost justified?

High reliability is not necessarily an end in itself but may be desirable in order to reduce the expected cost due to subsystem failure. However, this may not be the wisest use of funds since the expected cost due to subsystem failure is not the only cost involved. The subsystem itself may be very costly. We should not consider either the cost of the subsystem or the expected cost due to subsystem failure separately but should minimize the total of the two costs, i.e., the total of the cost of the subsystem plus the expected cost due to subsystem failure.

This final report discusses the Combined Analysis of Reliability, Redundancy, and Cost (CARRAC) methods which were developed under Grant Number NAG3-1100 from the NASA Lewis Research Center. CARRAC methods and a CARRAC computer program employ five models which can be used to cover a wide range of problems. The models contain an option which can include repair of failed modules.

## ASSUMPTIONS AND NOTATION

In this paper assume perfect switching devices (if needed) of negligible cost and independence of the subsystem modules.

NOTATION

n	number of modules in the subsystem
k	minimum number of good modules for the subsystem to be good
r	reliability of the whole system for other than failure of the subsystem
$r_s$	reliability of the subsystem
$c_1$	loss due to failure of the subsystem
$c_2$	loss due to subsystem output at $v_c$ (for models 3, 4, and 5)
$c_3$	cost of a one module subsystem capable of full output
$c_4$	cost of a module in a k-out-of-n:G subsystem when k is fixed (see later discussion)
$c_6$	cost to repair a module

$g(k)$  function which relates cost of subsystem to the number of modules in the subsystem  
 $v_c$  fraction of subsystem output necessary so that the mission is not a failure  
 $p$  probability that a module is good  
 $q$  probability that a module fails or  $1-p$   
 $C$  the total of the cost of the subsystem itself plus the expected loss due to subsystem failure  
 $\lambda$  failure rate of a module (models 4 and 5 and repairs)  
 $T_0$  mission time  
 $\mu_r$  the mean time to repair a module  
 $\sigma_r$  the standard deviation of the time to repair a module

## INTRODUCTION

Since expected value is an important ingredient in our quest for finding the best subsystem, consider the expected cost due to subsystem failure, denoted as  $E\{\text{cost due to subsystem failure}\}$ . As with all expected values, it depends upon both the dollar cost and the probability of its occurrence. Let  $c_1$  be the cost due to failure of the subsystem, including all costs incurred by subsystem failure (but not the cost of the subsystem itself). This number could be the entire cost of the main system (or even greater) if failure of the subsystem resulted in failure of the main system. In other instances  $c_1$  would be less than the cost of the main system, e.g., failure of the subsystem resulted in only a partial failure of the main system.

Now the expected cost due to subsystem failure is  $c_1$  times the probability that this cost will be experienced. The only time that this cost will be experienced is if both the subsystem fails and the main system does not fail. If the main system fails, then we will not experience a subsystem failure. For example, if we're considering a power subsystem in a rocket, the rocket may explode on the launch pad due to a fuel problem. Even if the power subsystem would have failed in flight, we would not experience this failure. Let  $r$  be the reliability of the main system (for other than failure of the subsystem) and let  $r_s$  be the reliability of the subsystem. [Note that  $\text{Pr}$  means "probability of". We will also use the fact that  $\text{Pr}(A \text{ and } B) = \text{Pr}(A \cap B) = \text{Pr}(A)\text{Pr}(B|A)$ .] Then

$$\begin{aligned}
 E\{\text{cost due to subsystem failure}\} &= c_1 \text{Pr}\{\text{subsystem failure} \cap \text{main system good}\} \\
 &= c_1 \text{Pr}\{\text{subsystem failure} | \text{main system good}\} \text{Pr}\{\text{main system good}\} \\
 &= c_1(1-r_s)r = rc_1(1-r_s).
 \end{aligned}$$

We can minimize this expected cost by building a subsystem with an extremely low probability of failure (high reliability). However, it is not clear that we should build the most reliable subsystem possible since this will minimize only the expected cost due to subsystem failure but does not consider the cost of building the subsystem. We should not consider the two costs separately. We therefore minimize the total of the two costs, i.e., the total of

the cost of the subsystem plus the expected cost due to subsystem failure. The total cost to be minimized is

$$\begin{aligned} C &\equiv \text{cost of the subsystem} + E(\text{cost due to subsystem failure}) \\ &= \text{cost of the subsystem} + rc_1(1-r_s) \end{aligned} \quad (1)$$

In minimizing cost C we see that we are balancing the cost of the subsystem and the expected cost due to subsystem failure.

### SELECTING THE BETTER SUBSYSTEM

Suppose that we are considering two subsystems. Subsystem 1, which costs \$200 has a .97 reliability. Subsystem 2, with a cost of \$100, has a .94 reliability. Without further analysis, there is no clear "best" subsystem and the choice is often based upon the amount budgeted for the subsystem.

Assume that the two subsystems under consideration will be part of a main system which has a reliability (exclusive of the subsystem under consideration) of  $r = .96$ . We'll further assume that failure of the subsystem will result in a cost of  $c_1 = \$10,000$ . Let us first compare the  $E(\text{cost due to subsystem failure})$  for each of the two subsystems.

$$\begin{aligned} \text{For subsystem 1,} \\ E(\text{cost due to subsystem failure}) &= rc_1 \Pr\{\text{subsystem failure}\} \\ &= rc_1(1-r_{s1}) \\ &= .96 \times \$10,000 \times .03 = \$288. \end{aligned}$$

For subsystem 2,

$$\begin{aligned} E(\text{cost due to subsystem failure}) &= rc_1(1-r_{s2}) \\ &= .96 \times \$10,000 \times .06 = \$576. \end{aligned}$$

Subsystem 2 has a higher expected cost than subsystem 1. However, since 2 is also less expensive, we need to compare the overall expected cost,  $C$ , for 1 and for 2.

$$\begin{aligned} \text{For subsystem 1,} \\ C_{s1} &= \$200 + \$288 = \$488. \end{aligned}$$

$$\begin{aligned} \text{For subsystem 2,} \\ C_{s2} &= \$100 + \$576 = \$676. \end{aligned}$$

Since  $C_{s1} < C_{s2}$ , we select subsystem 1 over subsystem 2.

For further information on expected values or on selecting the best subsystem, see [3].

## K-OUT-OF-N:G SUBSYSTEMS

In this article we'll direct our attention to a specific type of subsystem, called a k-out-of-n:G subsystem. Such a subsystem has n modules, of which k are required to be good for the subsystem to be good. As an example consider the situation where the engineer has a certain power requirement. He may meet this requirement by having one large power module, two smaller modules, etc. The number of modules required is called k. For example, the engineer may decide that  $k = 4$ . Then each module is  $1/4$  of the full required power. Therefore, the subsystem must have 4 or more modules for the full required power. The number of modules used in the subsystem is called n. For example, an  $n = 6$  and  $k = 4$  subsystem would have 6 modules each of  $1/4$  power and thus would have the output capability of 1.5 times the required power. The engineer chooses n and k. Selection of the different values of n and k results in different subsystems, each with different costs and reliabilities. Since each n and k yields different subsystems with different costs, we can choose the subsystem (the n and k) which will minimize cost C.

### MODEL 1

The simplest k-out-of-n: G model is one where the modules are independent and all have common probability of being good p and common probability of failure  $q = 1-p$ . Let X count the number of good modules. Now

$$\begin{aligned} E(\text{cost due to subsystem failure}) &= rc_1 \Pr(\text{subsystem failure}) \\ &= r c_1 \Pr(X < k) = r c_1 \sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x}. \end{aligned} \quad (2)$$

Recall that  $C = \text{cost of subsystem} + E(\text{cost due to subsystem failure})$ . We therefore need to also consider the cost of the subsystem. First consider a simple situation where k is fixed. Here we are free to choose n. Then  $n-k$  will be the redundancy or number of spares in the subsystem. If each module costs  $c_4$ , then the cost of subsystem =  $nc_4$ . Using this with (2) we obtain

$$\begin{aligned} C &= \text{cost of subsystem} + E(\text{cost due to subsystem failure}) \\ &= n c_4 + r c_1 \sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x}. \end{aligned}$$

We wish to find the n which minimizes cost C.

The author has written a program (QuickBASIC 4.5) called CARRAC to find the n which minimizes C. Additionally this program will,

if you desire, graph  $C$  as a function of either  $p$  or  $c_1$ . CARRAC will plot the best subsystems (i.e. the ones with the lowest  $C$ 's) over ranges of  $p$  or  $c_1$ . This allows you to not only select the best subsystem for a particular value of  $p$  or  $c_1$  but also to view what happens to  $C$  for nearby values of  $p$  or  $c_1$ .

As an example, consider the situation when  $k = 1$ , where only one module is required to be operational for the subsystem to be operational. The reliability of this single module is estimated to be .95 ( $p = .95$ ). Let the reliability of the system for other than failure of the subsystem be .9, ( $r = .9$ ). The cost of one module is 1 ( $c_4 = 1$ ) million dollars (throughout the remainder of the paper all costs will be in millions of dollars). The cost due to failure of this subsystem is 10 ( $c_1 = 10$ ).

Figure 1 shows a plot of  $C$  for  $p$  ranging from .79 to .99 and  $n$ 's of 1 through 4. When the reliability of a single module  $p = .95$ ,  $n = 1$  has the lowest value of  $C$ . Therefore the best subsystem in this case is one with no spares. We see from figure 1 that the  $n = 1$  subsystem (no spares) has the lowest value of  $C$  for any  $p > .87$ . If  $p < .87$ , then  $n = 2$  (one spare) has the lowest value of  $C$ . For  $p < .79$ , we would view the graph over the range of  $p < .79$ .

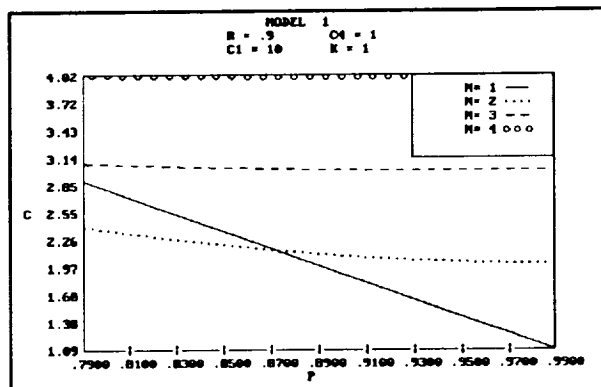


Figure 1

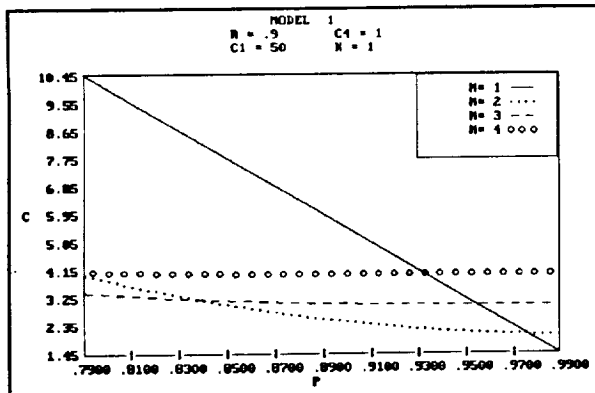


Figure 2

Now suppose instead that  $c_1$  (cost due to failure of the subsystem) is 50. Figure 2 shows the plot of  $C$  for  $c_1 = 50$ . We first note that if  $p = .95$ , then the  $n = 2$  subsystem is the best. Comparing figures 1 and 2 (at  $p = .95$ ) we see that the larger value of  $c_1$  (in figure 2) requires a larger value of  $n$ . This principle holds in general. If the cost of subsystem failure increases then more redundancy is required. If  $.83 < p < .98$ , figure 2 shows that

the  $n = 2$  subsystem is best. If  $p$  is below .83 then more redundancy ( $n=3$ ) is required. If  $p > .98$ , then no redundancy ( $n=1$ ) is required.

## MODEL 2

If, in model 1, we are also free to choose  $k$  in our subsystem, then we have model 2. Let  $c_3$  be the cost of a subsystem consisting of exactly one module. Further suppose that the cost of a subsystem with exactly  $k$  modules is  $c_3 g(k)$ . Here  $g(k)$  is the factor which measures the (generally) increased cost of building a subsystem consisting of  $k$  smaller modules rather than one large module. If  $g(k) = 1$  for all  $k$ , then a subsystem of  $k$  modules costs the same as a subsystem consisting of a single module. Any  $g(k)$  may be used. For example, if a subsystem of 2 smaller modules costs 4 times as much as a single module subsystem then  $g(2) = 4$ . Therefore this subsystem would cost  $c_3 g(2) = 4c_3$ . If a subsystem of 3 smaller modules costs 7 times as much as a single module subsystem then  $g(3) = 7$ . Other values for  $g(k)$  may be defined in a similar manner. Therefore, in the above example,  $g(1) = 1$ ,  $g(2) = 4$ ,  $g(3) = 7$ , etc. We also assume that each module in the subsystem costs  $c_3 g(k)/k$ , which is  $1/k$  of the total cost for  $k$  modules. Since we have a total of  $n$  modules in the subsystem, then the cost of the subsystem  $= nc_3 g(k)/k$ . Using this with (2) we obtain

$$C = \text{cost of subsystem} + E(\text{loss due to subsystem failure})$$

$$= n c_3 g(k) / k + r c_1 \sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x}.$$

For any particular situation with given values of  $c_1$ ,  $c_3$ ,  $r$ ,  $p$  and  $g(k)$  we use CARRAC to select the  $n$  and  $k$  to minimize  $C$  as given above. There are two options for  $g(k)$  built into CARRAC. You may choose either  $g(k) = (1+b)g(k-1)$  or  $g(k) = k(1/k)^c$ , where you are free to set  $b$  or  $c$ .

If you believe that the cost of building a subsystem of  $k$  modules increases (or decreases) linearly with  $k$ , then you would choose the first option  $g(k) = (1+b)g(k-1)$ , with  $b > 0$  ( $b < 0$ ). For example, if building a subsystem of two smaller modules costs 20% more than building a single module subsystem, 3 modules costs 20% more than a subsystem of two modules, etc., then let  $b = .2$ . If you believe that the cost of building a subsystem is exponentially proportional to the number of modules in the subsystem then you would choose the second option  $g(k) = k(1/k)^c$ . For example, consider building a space electrical power subsystem. A rough rule of thumb says that the cost of smaller modules for a space electrical power subsystem is proportional to the electrical power raised to the .7, i.e.,  $g(k) = k(1/k)^{.7}$ . Therefore, a subsystem consisting of a single module capable of full power costs  $c_3 g(1) = c_3 1(1/1)^{.7} = 1.0c_3$ . A subsystem

consisting of 2 modules, each of 1/2 power, costs  $c_3 g(2) = c_3 2(1/2)^{.7} = 1.23c_3$  to build, etc. An  $n = 3$  and  $k = 2$  subsystem, (one having 3 modules each of 1/2 power) costs  $nc_3 g(k)/k = 3c_3(1/2)^{.7}/2 = 3c_3 \times 1.23/2 = 1.85c_3$  to build.

As an example of model 2, suppose we are building a space electrical power subsystem. The cost due to subsystem failure,  $c_1$ , is 240. Let the reliability of the system for other than failure of the subsystem be .9 ( $r = .9$ ). Suppose that the cost of building a single module capable of full power is 1 ( $c_3 = 1$ ). Using the rule of thumb stated above, we use the option for  $g(k)$  with  $c = .7$ . All of the above values are entered into CARRAC as parameters. An estimate of  $p$ , the reliability of an individual module, is .96. If we are unsure of this estimate, we can use CARRAC to view (figure 3) the best subsystems over  $p$  ranging from .89 to .99.

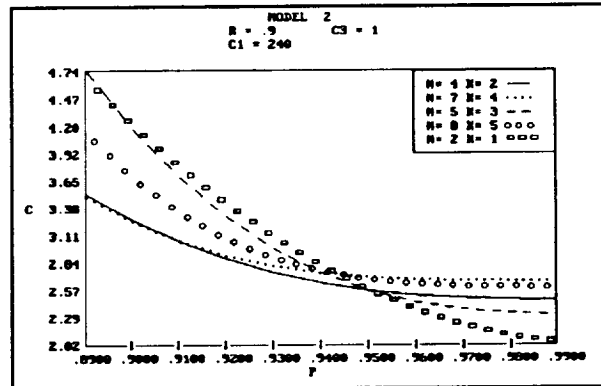


Figure 3

From figure 3, at  $p = .96$ , the  $n = 2, k = 1$  subsystem is best (lowest value of  $C$ ). If  $p < .95$ , the  $n = 4, k = 2$  subsystem is best. Note this is a flatter curve over the range of  $p$ , indicating a low value for  $C$  over a wide range of  $p$ .

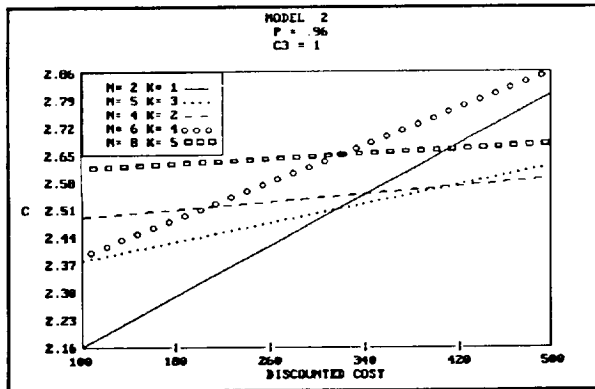


Figure 4

For the same example, suppose we wish to view what happens to  $C$  as  $c_1$  varies. Figure 4 (from CARRAC) shows, if  $c_1$  is below 310, then the  $n = 2, k = 1$  subsystem is best. However, for  $310 < c_1 < 400$ , the  $n = 5, k = 3$  subsystem is the best. For  $c_1 > 400$  the  $n = 4, k = 2$  subsystem is the best. This type of analysis could be used whenever you are unsure of  $c_1$  and wish to consider results over a range of values.

### MODEL 3

Figure 5 shows the loss due to subsystem failure, where  $v$  is the ratio of the actual output of the subsystem to the specification output. If  $v$  drops below some critical value  $v_c$ , the mission is a complete failure and the loss is  $c_1$ . However, if  $v$  is at  $v_c$ ,

then the loss is only  $c_2$ . As  $v$  increases above  $v_c$ , this loss decreases until there is no loss at full output.

Although  $h$  is linear in figure 5 other loss functions, e.g., a decreasing multi-step function, are appropriate. If  $h(v) = a - av$ ,  $v_c < v < 1$ ,  $a = c_2/(1-v_c)$ , (1) becomes

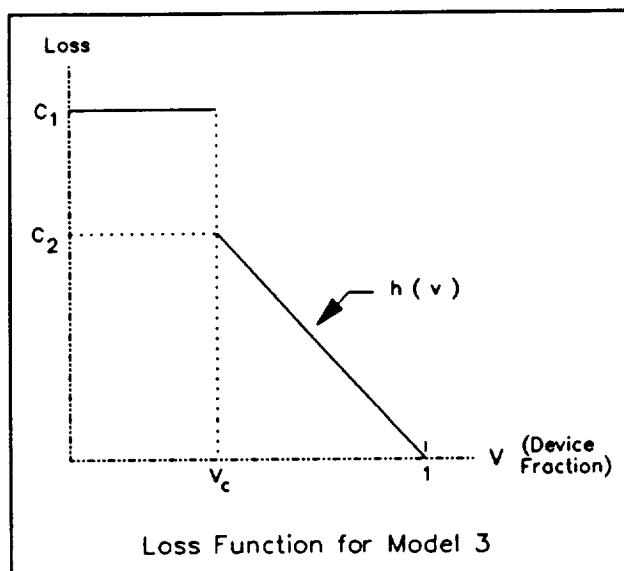


Figure 5

$$C = nc_3 g(k) / k + r c_1 \sum_{x=0}^{x < kv_c} \binom{n}{x} p^x q^{n-x} + r \sum_{x \geq kv_c}^{k-1} \binom{n}{x} p^x q^{n-x} (a - ax/k).$$

The third term on the rhs is expected loss due to partial failure of the subsystem. Again we can find, by means of CARRAC, the  $n$  and  $k$  which minimize  $C$ .

#### MODEL 4

Suppose in model 3 (with  $c_1 = c_2$ ) that mission time is also important. If modules fail exponentially with failure rate  $\lambda$ , then the probability of a module still operating successfully at time  $t$  is  $\exp(-\lambda t)$ . Let  $f(x, t)$  be the joint probability density function of  $x$  successes ( $n-x$  failures) and time  $t$ . Note that  $g(x)$  is the probability that, at time  $T_0$ , exactly  $x$  modules will be operating successfully.



$$\begin{aligned}
f(t, x) &= n \binom{n-1}{x} [\exp(-\lambda t)]^x [1 - \exp(-\lambda t)]^{n-x-1} \lambda \exp[-\lambda t] [\exp(-\lambda(T_0 - t))]^x \\
&= \frac{n!}{x!(n-x-1)!} [\exp(-\lambda T_0)]^x \lambda \exp(-\lambda t) [1 - \exp(-\lambda t)]^{n-x-1} \\
&\quad 0 < t < T_0, \quad x=0, 1, \dots, n-1.
\end{aligned}$$

$$\text{Now } g(x) = \int_0^{T_0} f(t, x) dt$$

$$\begin{aligned}
&= \binom{n}{x} [\exp(-\lambda T_0)]^x [1 - \exp(-\lambda T_0)]^{n-x} \quad x=0, 1, \dots, n-1 \\
&\text{with } g(n) = \exp[-\lambda T_0]^n.
\end{aligned}$$

If the output fraction is  $v_c$  at the start of the mission, our loss is  $c_2$ . As  $v$  increases above  $v_c$ , then this loss decreases until there is no loss at full output. With output at or above  $v_c$ , losses decrease with increasing time until there is no loss beyond mission time  $T_0$ . Additionally, for any given  $t$ ,  $h(v, t)$  decreases as  $v$  increases above  $v_c$ .

Consider now a general loss function  $h(v, t)$  [not necessarily the one illustrated by figure 6]. Again, for a given  $t$ ,  $h$  takes on values only for  $v = x/k$ . Now (1) becomes

$$\begin{aligned}
C &= nc_3 g(k) / k + r \sum_{x=0}^{k-1} \int_0^{T_0} h(x/k, t) f(x, t) dt. \\
\text{Let } h(x/k, t) &= d(x/k) \sum_{j=0}^m b_j t^j.
\end{aligned} \tag{3}$$

Then, after integrating, (3) becomes

$$\begin{aligned}
C &= nc_3 g(k) / k + r \sum_{x=0}^{k-1} d(x/k) (n-x) \binom{n}{x} [\exp(-\lambda T_0)]^x \lambda \\
&\quad \sum_{j=0}^m b_j \sum_{i=0}^{n-x-1} (-1)^{i+1} \binom{n-x-1}{i} [\lambda(i+1)]^{-(j+1)} \\
&\quad [1 - \exp[-\lambda(i+1)T_0]] \sum_{w=0}^j w! \binom{j}{j-w} [\lambda(i+1)]^{j-w} T_0^{j-w}
\end{aligned} \tag{4}$$

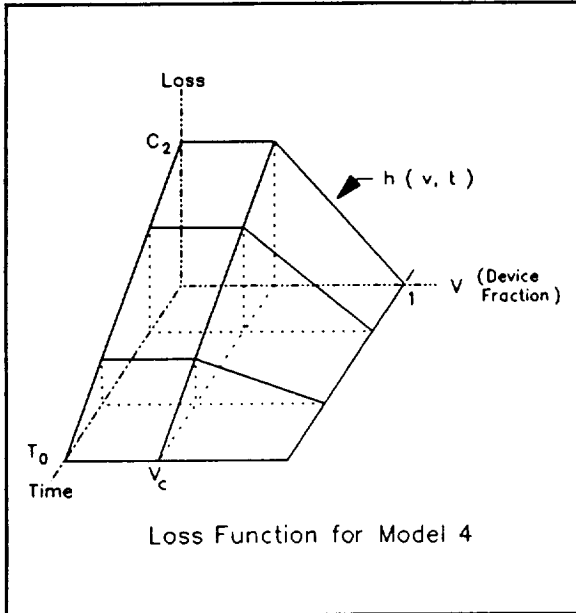


Figure 6

We wish to find the  $n$  and  $k$  which minimize  $C$ . Minimizing  $C$  in (4) is appropriate for any loss function,  $h(\cdot)$ , of the form given in (3). Using the loss function given in figure 6, for  $0 \leq x < kv_c$ ,  $d(x/k) = 1$ ,  $m = 1$ ,  $b_0 = c_2$  and  $b_1 = -c_2 T_0$ . For  $kv_c \leq x \leq k-1$  we have  $d(x/k) = 1 - x/k$ ,  $m = 1$ ,  $b_0 = a$  and  $b_1 = -aT_0$  where  $a = c_2(1-v_c)$  with  $0 < v_c < 1$ .

$$\begin{aligned}
 \text{Let } w_1(x) &= \binom{n}{x} (n-x) \exp[-\lambda T_0] x \lambda \\
 w_2(x) &= \sum_{i=0}^{n-x-1} (-1)^i \binom{n-x-1}{i} [\lambda(i+1)]^{-1} [1 - \exp[-\lambda(i+1)T_0]], \\
 w_3(x) &= \sum_{i=0}^{n-x-1} (-1)^i \binom{n-x-1}{i} [\lambda(i+1)]^{-2} \\
 &\quad [1 - \exp[-\lambda(i+1)T_0] - \lambda(i+1)T_0 \exp[-\lambda(i+1)T_0]].
 \end{aligned}$$

Using (4) we obtain

$$C = nc_3 g(k) / k + r \sum_{x=0}^{x < kv_c} c_1 w_1(x) [w_2(x) - T_0^{-1} w_3(x)] \\ + r \sum_{x \geq kv_c}^{k-1} [1 - x/k] a w_1(x) [w_2(x) - T_0^{-1} w_3(x)].$$

#### MODEL 4 APPLICATIONS

Model 4 might reasonably be applied to non-recoverable systems which, at the end of their service life, have no intrinsic or salvage value or which are prohibitively expensive to recover. Examples include undersea sonar systems anchored in deep water, instrument/telemetry packages located in remote regions or communications satellites in geosynchronous orbit. For a geosynchronous communications satellite a number of subsystems could be chosen as an example. Let us examine the satellite power system which can be divided into smaller identical modules. We again use the rule of thumb which says that the cost of a space power subsystem is proportional to the electrical power raised to the .7 ( $g(k) = k(1/k)^{.7}$ ). Suppose that the mission life is 7 years and the reliability of the satellite (exclusive of the power subsystem) over the mission life is .90. Because the satellite needs power for stationkeeping, computers and cooling, at least 10% of the specification power is needed for the satellite to survive. Therefore,  $v_c$  is 0.1. The satellite generates \$2 million per month revenue. In the event of satellite failure, a new satellite could be launched within two years at a cost of \$115 million. Therefore  $c_1$  (or  $c_2$ ) = 163 (115 plus 48 in lost revenue). Here we will assume that revenue is roughly proportional to power, i.e., if a module of the power subsystem fails, then one or more channels are no longer available. We estimate  $\lambda$  as  $3.5(10^{-6})$  and again use CARRAC to view  $C$  over a range of  $\lambda$  from  $1(10^{-6})$  to  $6(10^{-6})$ . Figure 7 shows the 5 best subsystems. For  $\lambda < 4(10^{-6})$  the  $n = 2, k = 1$  subsystem is optimal. For  $\lambda > 4(10^{-6})$ , the  $n = 3, k = 1$  subsystem is optimal.

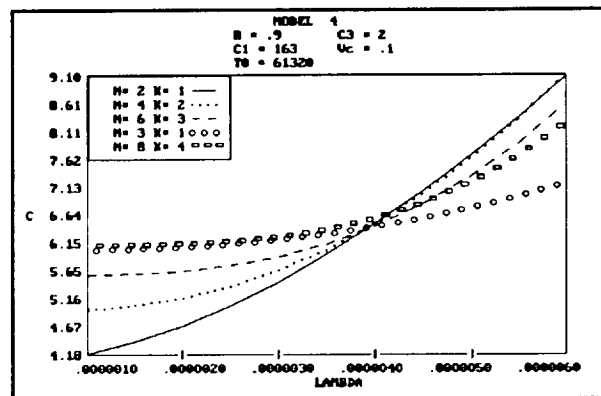


Figure 7

## MODEL 5

Suppose we have a situation similar to model 4 but now assume a loss of  $c_1$  if the output fraction from the subsystem is below  $v_c$  anytime during the life of the mission.

Model 5 could be applied to recoverable systems, systems which have inherent salvage value or manned systems. Examples include manned aircraft or spacecraft, recoverable undersea vehicles or spacecraft.

Model 5 implies that if the output fraction of the subsystem falls below the critical value  $v_c$ , something catastrophic will occur, such as loss of the whole system or loss of life.

With these systems, loss or significant degradation of a critical subsystem might cause loss of the craft and occupants.

An example of such a loss function is given by figure 8.

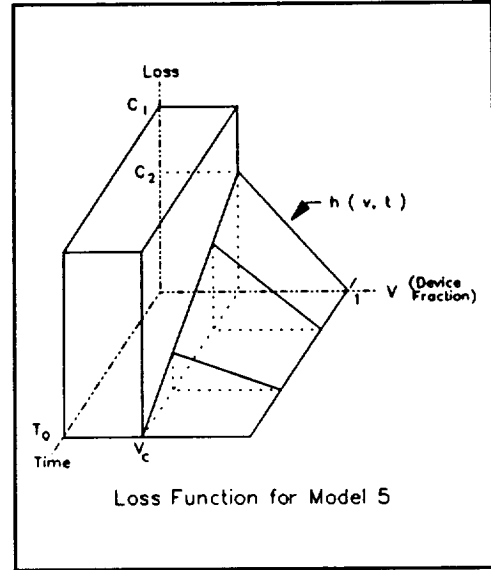


Figure 8

With this loss function, for  $x < kv_c$ ,  $b_0 = c_2$  and  $b_1 = 0$  and for  $kv_c \leq x \leq k-1$ , we have  $d(x/k) = 1 - x/k$ ,  $m = 1$ ,  $b_0 = a$  and  $b_1 = -aT_0^{-1}$  where  $a = c_2 (1 - v_c)^{-1}$  with  $0 < v_c < 1$ .

Using (4)

Use of CARRAC is applicable to view  $C$  over a range of either  $\lambda$  or

$$C = nc_3 g(k) / k + r \sum_{x=0}^{k-1} c_1 g(x)$$

$$+ r \sum_{x \geq kv_c}^{k-1} [1 - x/k] a w_1(x) [w_2(x) - T_0^{-1} w_3(x)].$$

## Repairability

Since we are considering repairs, we must now consider the useful time of the subsystem or the mission time,  $T_0$ . Therefore  $p$ , the probability that a module is good, is a function of  $T_0$ .

If we assume that failures occur at random, i. e. exponentially, then  $p = \exp(-\lambda T_0)$ , where  $\lambda$  is the failure rate. We further assume that repairs are equivalent to replacement, i. e., a repair to a module will result in a module as good as new. We also assume that the time to repair is normally distributed, with an estimated mean,  $\mu_r$  and standard deviation  $\sigma_r$ .

For all repair situations, analysis has been done in CARRAC by means of simulation. For this reason, if you have a situation where repair is an option, the analysis to find the optimal subsystem may require considerable computer time. The required time depends upon both the subsystems being considered and the speed of the computer. If you are running the analysis for a particular subsystem, e. g.,  $n = 7$  and  $k = 4$ , the amount of time required for simulating repairs is usually quite short, in the range of a minute or so. However, if you request a search and graphical analysis, then the simulation may require several hours. CARRAC also allows you to choose low, medium or high resolution for the simulation. High resolution has the most accurate results but is also the slowest. Medium and low are faster but with correspondingly less accurate results. You might consider low resolution for your initial searches and increase the resolution as you approach the optimum.

#### **Repair: models 1 and 2**

The scenarios for models 1 and 2 are identical. Since we are using simulation, we have a number of trials. Consider the first trial. If we let  $s$  be the number of good modules in the subsystem at a given time, then  $s = n$  at the beginning of the mission. If a module fails, then  $s = n-1$ . If  $s < k$ , then the subsystem fails and we incur a cost of  $rc_1$  (due to the loss of the entire subsystem). If  $s \geq k$ , we initiate repair on the first module and a cost of  $c_6$  (the cost of repairing one module) is incurred (Again, the amount of time required for repair is a normally distributed random variable with mean  $\mu_r$  and standard deviation  $\sigma_r$ ). If the failed module is repaired before another module fails, then our total cost up to this time is  $c_6$ . If another module fails before repair is completed on the first module, then  $s = n-2$ . If  $s < k$ , then the subsystem fails and we incur a cost of  $rc_1$  (due to the loss of the entire subsystem). Therefore, our total cost for the first trial is  $rc_1 + c_6$ . If  $s \geq k$ , we initiate repair on the second module and incur another cost of  $c_6$ . If, throughout the entire mission  $s \geq k$ , then the subsystem has not failed and our total cost involves only repair costs, the number of failed modules times  $c_6$ . If, however, at some time during the mission  $s < k$ , the subsystem has failed and we incur a cost of  $rc_1$  due to failure of the subsystem. Therefore our total cost for the first trial is  $rc_1$  plus the number of failed modules times  $c_6$ . We repeat this a large number of times (depending upon the level of resolution chosen) and average our costs over all trials. The cost  $C$  is given by

$C \equiv \text{cost of the subsystem} + E\{\text{cost due to subsystem failure}\}$   
 $+ E\{\text{cost of repair}\}.$

#### Repair: models 3,4 and 5

The situation for model 3 differs in that we allow for partial failure of the subsystem, according to figure 5. We assume that complete failure of the subsystem results in a loss of  $rc_1$ , regardless of the time (into the mission) at which complete failure of the subsystem occurs. For model 4 (see fig. 8), we assume that the cost of complete failure of the subsystem,  $rc_1$ , is weighed by the proportion of the mission time over which complete failure occurs. For example, if the mission time is 1000 hours and complete failure of the subsystem occurs at 900 hours, the cost of complete failure is  $.1 rc_1$ .

Let's consider how these costs are calculated.

If, in the first trial,  $s \geq k$  throughout the entire mission, then the subsystem has not failed, even partially, and our total cost involves only repair costs. Therefore our total cost for the first trial is the number of failed modules times  $c_6$ . If  $s \geq kv_c$  throughout the entire mission, then the subsystem has not completely failed and our total cost involves only repair costs and the cost due to partial failure (which is weighed by the amount of time that the subsystem is in the particular state of partial failure). Therefore our total cost for the first trial is the number of failed modules times  $c_6$  plus the costs associated with partial failure. If, however,  $s < kv_c$  at some time during the mission, the subsystem has failed and we incur a loss due to complete failure of the subsystem. Models 3 and 4 differ here in the loss assigned to the complete failure of the subsystem,  $E\{\text{cost due to subsystem failure}\}.$

For model 3, the loss assigned to complete failure of the subsystem is  $rc_1$ . Therefore our total cost for the first trial is  $rc_1$  plus the number of failed modules times  $c_6$  plus the costs associated with partial failure. We repeat this a large number of times and average our costs over all trials.

For model 4, the loss assigned to complete failure of the subsystem is  $rc_1$  weighed by the proportion of mission time remaining. Therefore our total cost for the first trial is  $rc_1$  weighed by the proportion of mission time remaining plus the number of failed modules times  $c_6$  plus the costs associated with partial failure. We repeat this a large number of times and average our costs over all trials.

Therefore, for either models 3 or 4, the cost  $C$  is given by

$C \equiv \text{cost of the subsystem} + E\{\text{cost due to subsystem failure}\}$   
 $+ E\{\text{cost of repair}\} + E\{\text{cost due to partial subsystem failure}\}.$

We remark that, if we allow repairs in model 3 and consider the mission time, then models 3 and 5 are identical.

#### CARRAC

It is anticipated that the CARRAC program (written in QuickBASIC) will become available in the future through NASA's Computer Software Management and Information Center (COSMIC).

#### SUMMARY

Table 1 contains a summary of the five models which can be applied in a redundancy cost analysis.

Table 1

##### Redundancy Cost Models Considered in this Paper

- |         |   |
|---------|---|
| Model 1 | Simplest cost model. The subsystem consists of $n$ modules, of which $k$ are required for success of the mission. If less than $k$ modules are good, a loss of $c_1$ occurs. In model 1, $k$ is fixed.  |
| Model 2 | Same as model 1 except $k$ may also vary. The $g(k)$ cost function is also available to be used where increased redundancy brings in more (non-linear) cost.  |
| Model 3 | Model 3 expands on models 1 and 2. Linear (or other) loss functions are utilized. If less than $k$ modules are good, some loss will occur but not necessarily the entire loss of $c_1$ . The loss which occurs depends upon some critical output fraction $v_c$ . |
| Model 4 | Model 4 considers time in the loss function. Modules in the subsystem fail exponentially with rate $\lambda$ .  |
| Model 5 | Model 5 handles situations where output fraction below $v_c$ causes a loss which is not time dependent, e.g., manned space missions where loss of a major portion of a critical subsystem may cause loss of life.   |

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